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DETERMINATION OF THE DIMENSIONS OF THE SATURATION ZONE FOR INFILTRATION FROM A CHANNEL WITH A SHALLOW WATER DEPTH

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A hydrodynamic solution has been considered [1-4] for the planar stationary case of a freshwater lens established by infiltration in accordance with Darcy's law from a channel involving the displacement of saline groundwater from the channel zone. It is assumed that the depth of water in the channel is infinitely shallow and that the flow factor that compensates for the loss from the channel is evaporation from the free surface. Here we examine infiltration from a channel into a layer of homogeneous isotropic soil of thickness T with a horizontal impermeable layer underneath. This case is a limiting one for the above problem when the density of the saline water increases without limit. The solution is found as in [1-4] by the method of [5], which is based on the analytical theory of ordinary differential equations. The canonical region is taken as the region for which the characteristics of the filtration flow can be derived in closed form in terms of certain special functions.

In view of the symmetry of the infiltration region we restrict consideration to the right-hand half, which is shown schematically in Fig. 1. The bottom of the channel is represented by a horizontal line of length $2l$. With the coordinate system shown in Fig. 1, we locate the plane of potential comparison in the plane y = 0, and then the following conditions are obeyed at the boundary of the infiltration region:

$$\begin{gathered} y=0,\ \varphi_r=0 \text{ on } AD,\ x=0,\ \psi_r=0 \text{ on } AB,\ y=T,\ \psi_r=0 \text{ on } BC, \\ \varphi_r+y=0,\ \psi_r+\varepsilon_r x=\varepsilon_r L \text{ on } CD, \end{gathered} \tag{1}$$

where $\omega_r = \varphi_r + i\psi_r$ is the complex filtration potential referred to the filtration coefficient, with φ_r the reduced potential for the filtration rate and ψ_r the reduced current function, while z = x + iy is the complex coordinate in the infiltration region and ε_r is the reduced evaporation rate.

As the auxiliary region we take half the plane of w in Fig. 2. In the method used here, the functions $d\omega/dw$ and dz/dw are unknowns to be determined as the solutions to a certain linear differential equation of the Fuchs class with regular singular points. We first consider the behavior of the functions $d\omega/d\zeta$ and $dz/d\zeta$, where ζ is the upper half-plane, and we find that the characteristic parameters of these functions near the singular points have the following values: near point A $(\zeta=-a)(-1/2,\ -1/2)$, near point B $(\zeta=0)(-1/2,\ 0)$, near point C $(\zeta=1)(\nu/2-1/2,\ -\nu/2-1/2)$, and near point D $(\zeta=\infty)(3/2,\ 2)$, where $\nu = 1-(2/\pi)\arctan\sqrt{\varepsilon_r}$, and the singularity at point $\zeta=-a$ can be eliminated.

The solution that satisfies the conditions of (1) takes the form

$$\frac{d\omega}{dw} = A\frac{\sqrt{\varepsilon_r}\,\mathrm{sh}\,\nu w}{\sqrt{\mathrm{sh}^2 w + a\,\mathrm{ch}^2 w}},\quad \frac{dz}{dw} = A\frac{\mathrm{ch}\,\nu w}{\sqrt{\mathrm{sh}^2 w + a\,\mathrm{ch}^2 w}}, \tag{2}$$

where A is some real constant.

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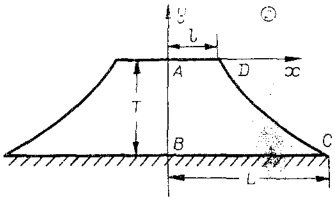

Fig. 1

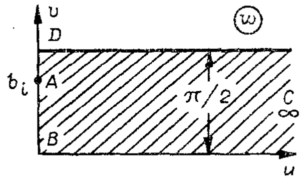

Fig. 2

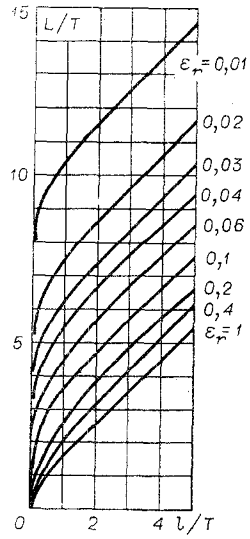

Fig. 3

We obtain the equation for the right branch of the free surface DC if we separate the real and imaginary parts in the second equation of (2) and then integrate them. This gives

$$x = l + A \sin \frac{\pi \nu}{2} \int_0^w \frac{\operatorname{sh} \nu t\, dt}{\sqrt{\operatorname{ch}^2 t + a \operatorname{sh}^2 t}},$$
$$y = -A \cos \frac{\pi \nu}{2} \int_0^w \frac{\operatorname{ch} \nu t\, dt}{\sqrt{\operatorname{ch}^2 t + a \operatorname{sh}^2 t}} \quad (0 \leqslant w \leqslant \infty). \tag{3}$$

We successively integrate the second of the expressions in (2) from point A to point D, from point B to point A, and from point B to point C, which gives correspondingly

$$l = \frac{A \cos b}{\sqrt{2}} \int_{2b}^{\pi} \frac{\cos \frac{\nu}{2} t\, dt}{\sqrt{\cos 2b - \cos t}}; \tag{4}$$

$$T = \frac{A\pi \cos b}{2} P_{\frac{\nu-1}{2}} (\cos 2b); \tag{5}$$

$$L = \frac{A\pi \cos b}{2 \cos \frac{\pi \nu}{2}} P_{\frac{\nu-1}{2}} (-\cos 2b), \tag{6}$$

where b is a constant related to a by $a = \tan^2 b$, $P_\nu(z)$ is a spherical Legendre function of the first kind with index ν [6], and $P_\nu(z) = F(-\nu, 1+\gamma, 1, (1-z)/2)$; $F(\alpha, \beta, \gamma, z)$ is hypergeometric function with parameters α, β, and γ.

From Eqs. (4)-(6) we constructed a graph for the function L/T with arguments l/T and ε_r (Fig. 3).* In conclusion we note that $l \to (2/\pi) \ln \tan (b/2)$ and $L \to \infty$ for $\varepsilon_r \to 0$.

We are indebted to Professor S. N. Numerov for a discussion of this study and his useful remarks.

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*First of all in (4) we make the substitution $t = \arcsin\sqrt{\sin^2 b + \cos^2 b \sin^2 \nu}$ to eliminate the singularity in the integrand at the lower limit of integration.

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INFILTRATION OF A SALT SOLUTION INTO A SWELLING SOIL

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The physicochemical and hydraulic parameters of soils are largely dependent on the content of clay minerals, which make up more than half of all sediments in the earth's crust. The lattice surfaces of the clay particles bear negatively charged oxygen ions, and therefore soils are basically cation-exchange materials capable of taking up cations from an electrolyte and exchanging equivalent amounts of positively charged ions. A dry clay soil on wetting by water or a solution swells and absorbs the water and solutes. The cause of the swelling is hydration of the ions in the hydrophylic groups in the soil. The extent of the swelling is dependent on the hydrated radius of the ion and the exchange capacity of the soil-absorbing complex. The swelling is accompanied by coalescence of colloidal particles, which leads to an increase in the amount of relatively immobile water [1, 2] and a substantial reduction in the filtration capacity of the soil.

The converse phenomenon is peptization or particle dispersion, which is accompanied by a reduction in the amount of bound water and improvement in the permeability, as is observed when an electrolyte infiltrates into a soil containing fresh water. Experiments in the field and in the laboratory with soil systems [3, 4] show that the balance between peptization and coalescence may produce large reductions or increases in the permeability of a given specimen of natural soil. It is important to consider these phenomena in developing methods of calculating the water-salt conditions in soils during irrigation and draining, as well as in research on the stability of earth dams and related problems.

Here we consider a model case of the infiltration of fresh water into a clay soil whose skeleton retains a certain amount of salt solution with a given concentration.

1. Formulation. The theory of double electrical layers implies that the concentration C_i in moles/liter of ion i in the solution surrounding the negatively charged surface of a clay particle is [5] given by

$$C_i = C_i^0 \exp[-z_i e(\psi - \psi^0)/kT],$$

where z_i is the valency of the ion, e is the electronic charge, $\psi = \psi(y)$ is the electrical potential, k is Boltzmann's constant, T is absolute temperature, C_i^0 and ψ^0 are the values of the equilibrium concentration and electrical potential, respectively, as measured far from the surface, and y is a coordinate measured along the normal to the surface of the particle. The solution in an elementary volume at a certain distance y will be attracted to the charged surface under the action of the electric field with a force

$$dp = -\sum_i z_i C_i \frac{N_A e}{1000}\, dy \frac{d\psi}{dy} = -\frac{N_A e}{1000} \sum_i z_i C_i^0 \exp\left[-\frac{z_i e(\psi - \psi^0)}{kT}\right] d\psi,$$

where N_A is Avogadro's number and the summation is taken over each ion ($z_i > 0$ for a cation and $z_i < 0$ for an anion). After integration of this expression with respect to p from p^0 to p and with respect to ψ from ψ^0 to ψ we get the value of the excess pressure (swelling pressure) $\Delta p = p - p_0$ acting near the surface of a colloidal particle:

$$\Delta p = \frac{RT}{1000} \sum_i (C_i - C_i^0), \qquad (1.1)$$

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